Functions of random variables, Expectation

Putting a value on random variables

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What we aim to model

“Random variable”: notes

Functions induce random variables

Numeric random variables

Distribution (graphical)

Examples: Bernoulli

Examples: Binomial

(Numerical) joint distributions

Marginal distributions

Independence

Expectation

Definition. *The expectation of a numeric random variable* $X = (\Omega, \mathbb{P})$ *is given by the following:*

$$\mathrm{E}[X] \coloneqq \sum_{x \in \Omega} x \mathbb{P}(X = x).$$

Linearity of expectation

Theorem. *Let* X, Y *be two (numeric) random variables, and their joint distribution given by* $\mathbb{P}(X = x, Y = y)$. *The expectation is a "linear operator": that is,*

1. $\mathrm{E}[X + Y] = \mathrm{E}[X] + \mathrm{E}[Y]$,
2. $\mathrm{E}[cX] = c\mathrm{E}[X]$ for any fixed $c \in \mathbb{R}$.

Linearity of expectation

Product of independent expectations

Theorem. *Let* X, Y *be two (numeric) random variables, and their joint distribution given by* $\mathbb{P}(X = x, Y = y)$. *If* X, Y *are independent, then expectation factors out their product:*

$$\mathrm{E}[XY] = \mathrm{E}[X]\mathrm{E}[Y].$$

Product of independent expectations